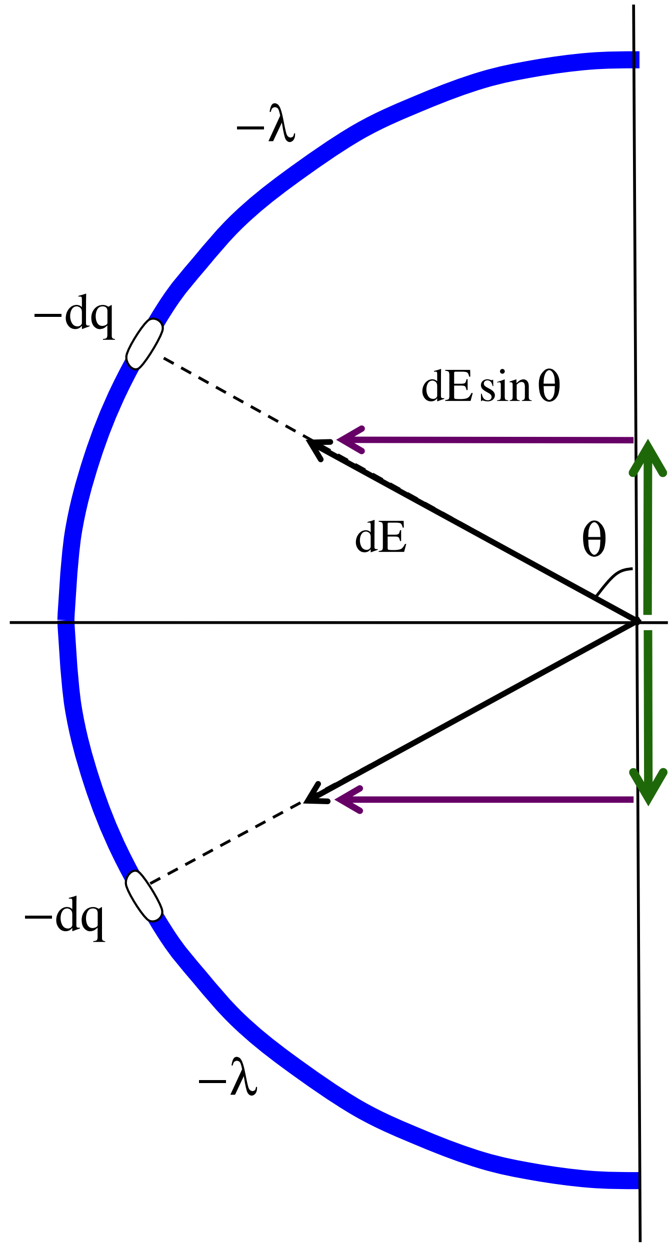
Problem 23.35

A uniform, negatively charged semi-circular hoop will have an electric field function derives as follows:

To begin with, notice that the electric field components from differential pieces of charge “-dq” on opposite sides of the hoop’s horizontal axis will have vertical components that add to zero (see sketch). That means we only need to determine the net field due to the horizontal components.



Using the added bit of trickery associated with the fact that a differential length "ds" on the curve is equal to:

$$ds = Rd\theta,$$

we can write the differential charge "dq" on that section as:

$$|dq| = |\lambda| ds = |\lambda| Rd\theta.$$

Noting also that

$$|\lambda| = \frac{Q/2\pi R}{2},$$

we can write:

$$|\vec{E}| = \int dE_y$$

$$= \int dE \sin\theta$$

$$= \int \left(k\frac{dq}{R^2}\right)\sin\theta$$

$$= \int \left(k\frac{|\lambda| Rd\theta}{R^2}\right)\sin\theta$$

$$= k\frac{\left(\frac{Q}{2\pi R/2}\right)}{R}\int_{\theta=0}^{\pi}\sin\theta\, d\theta$$

$$= k\frac{Q}{\pi R^2}(-\cos\theta)\Big|_{\theta=0}^{\pi} = k\frac{Q}{\pi R^2}\left[(-\cos\pi)-(-\cos 0^\circ)\right]$$

$$= 2k\frac{Q}{\pi R^2}$$

$$= 2\left(9\text{x}10^9 \text{ N}\bullet\text{m}^2/\text{C}^2\right)\frac{\left(7.50\text{x}10^{-6} \text{ C/m}\right)}{\pi(.140 \text{ m})^2}$$

$$= 2.19\text{x}10^6 \text{ N/C}$$

(this will be in the $-\hat{i}$ direction)